# PROGRESS TOWARDS A PREDICTIVE MODEL OF THE LH-TRANSITION

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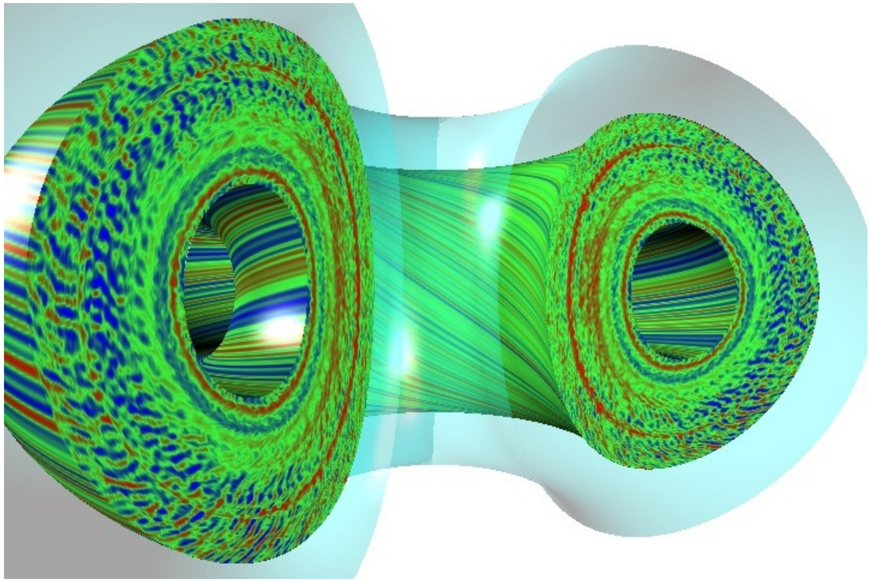

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# OUTLINE

- **Discovery of H-mode**
  - Importance of the plasma-wall boundary
  - Transport barrier
  - Bifurcation
- **First theories of H-mode**
  - Orbit loss at the boundary
  - Momentum bifurcation
  - The boundary layer
  - ExB velocity shear decorrelation
  - Energy transport bifurcation

# OUTLINE cont.

- **Numerical simulations with ExB shear**
  - Linear stability
  - Turbulence
  - Spectral shift model
  - Momentum transport

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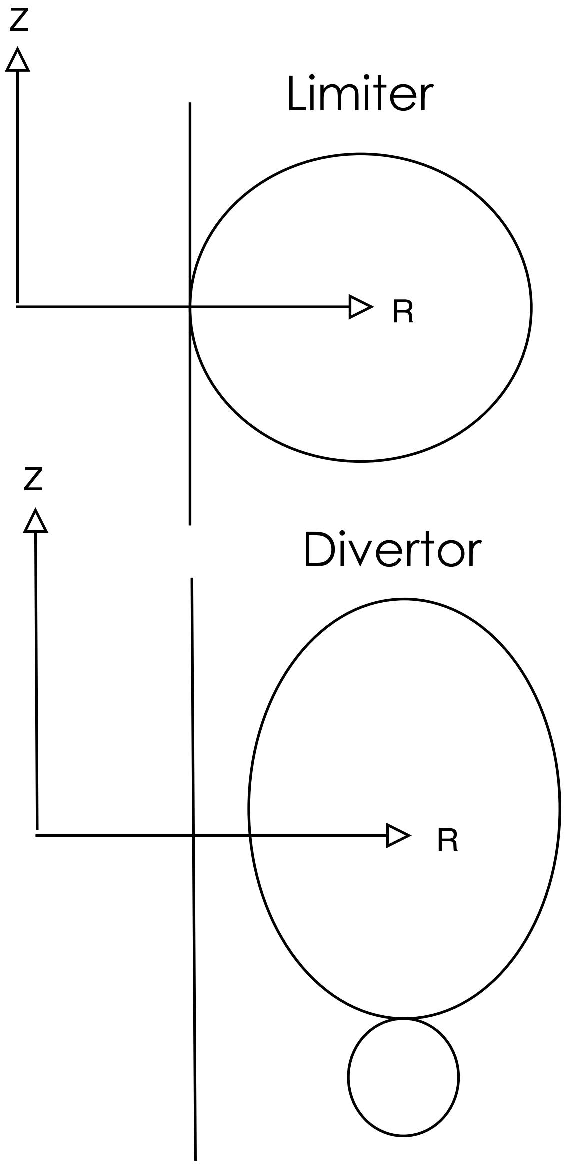
- **Mean fields and fluctuations**
  - Separation between driftwave turbulence and transport space and time scales
  - Zonal fluctuation mediated saturation and the Dimits shift
- **Predator-prey turbulence saturation model**

# OUTLINE cont.

- **Triggers & Dithers**
  - CER data finds a trigger event
  - DBS movies of dithering transitions
  - Kim-Diamond model of dithering LH-transitions
  - Mean field transport limit cycle oscillations
- **Progress towards a predictive model of the LH-transition**

# DISCOVERY OF H-MODE

- **1982 F. Wagner et al., Phys. Rev. Lett. 49, 1408.**
  - The High mode (H-mode) was discovered on the ASDEX tokamak in 1982 shortly after a closed divertor was installed.
  - H-mode has a “Transport barrier” in a narrow layer inside separatrix.
  - Hysteresis of the power for H/L and L/H transitions: bifurcation
  - Threshold power depends on divertor location.
  - Threshold power is independent of current: not a resonant q effect
  - H-mode energy confinement ~2x L-mode:  scales the same way.



# FIRST THEORIES OF H-MODE

- **Foundational concepts**
  - Orbit loss at the boundary
  - Momentum bifurcation
  - The boundary layer
  - ExB velocity shear decorrelation
  - Energy transport bifurcation

# FIRST THEORIES OF H-MODE: Orbit loss at boundary

- **1988 S.I. Itoh and K. Itoh, Phys. Rev. Lett. 60, 2270.**
  - Particle transport bifurcation theory.
  - Non-ambipolar particle flux is multi-valued in radial electric field: S-curve bifurcation.
  - Analogous to the electron and ion branches of radial electric field in stellarators.
- **1989 K.C. Shaing and E.C. Crume Jr., Phys. Rev. Lett. 63, 2369.**
- **1989 M. Tendler and V. Rozhansky, Bull. Am. Phys. Soc. 34, 2162.**
  - Bifurcation of the momentum equilibrium driven by ion orbit losses.

# FIRST THEORIES OF H-MODE: Momentum bifurcation

- **The collisional damping rate $\nu_\theta$ of poloidal equilibrium flow $U_\theta$ is reduced at high poloidal Mach number.** $M_\theta = \frac{BU_\theta}{B_\theta c_s}$ **"Stringer spin up"**
  - 1969 T.E. Stringer, Phys. Rev. Lett. 22, 770.

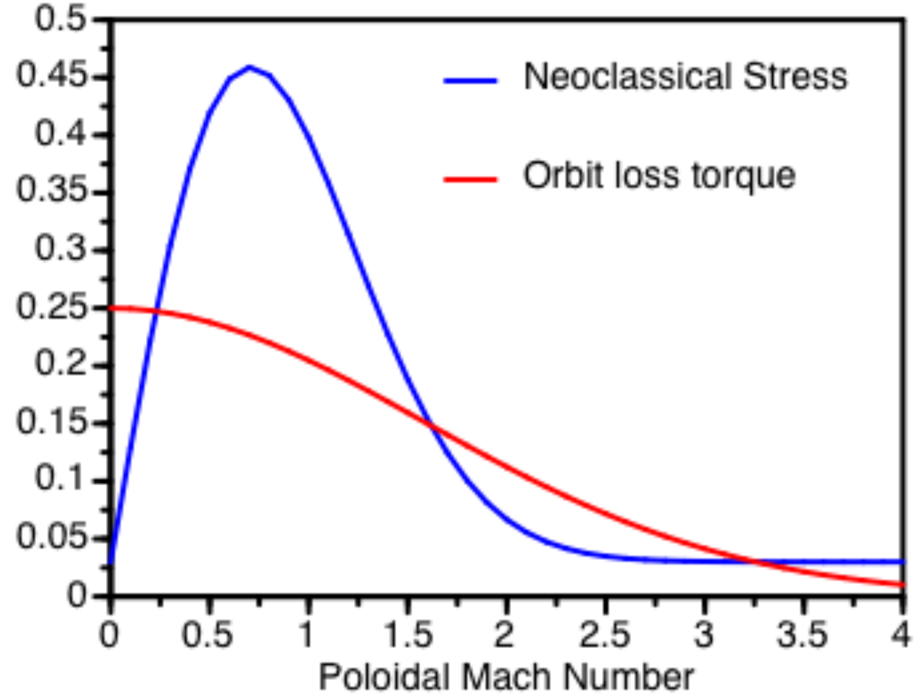
$$\frac{\partial}{\partial t} mnU_{\parallel} + \frac{\partial}{\partial r}\Pi_{\parallel} + mn\nu_\theta\left(U_\theta\right)\left(U_\theta - U_\theta^{neo}\right) = 0$$

$$\frac{\partial}{\partial t} mnU_{\varphi} + \frac{\partial}{\partial r}\Pi_{\varphi} - \frac{B_\theta J_r^{orbit}}{c} = 0$$

Steady state assuming $\Pi_\varphi \approx \Pi_\parallel$

$$mn\nu_\theta\left(U_\theta\right)\left(U_\theta - U_\theta^{neo}\right) = \frac{B_\theta J_r^{orbit}}{c}$$

The orbit loss driven bifurcation was disproved by CER data that showed the poloidal velocity changes in the opposite direction at the LH-transition



# FIRST THEORIES OF H-MODE: The boundary layer

- **1988 F.L. Hinton and G.M. Staebler, Nuclear Fusion 29 (1989) 405**
  - Scrape off layer (SOL) flows and ExB drift could explain why the LH threshold depends upon the divertor location.
- **1992 V. Rhozhansky and M. Tendler, Phys. Fluids B4, 1877**
  - Radial electric field in the SOL is positive due to sheath. $E_r \approx -2.8\frac{\partial T_e}{e\partial r}$
  - Poloidal flow in SOL is **not neoclassical**
  - A **viscous boundary layer** is needed inside the separatrix to transition from the SOL poloidal flow to the neoclassical solution in the core.

$$-\chi_\theta \frac{\partial^2 U_\theta}{\partial r^2} + \nu_\theta \left( U_\theta - U_\theta^{neo} \right) = 0$$

Layer width is $\lambda_\theta \approx \sqrt{\frac{\chi_\theta}{\nu_\theta}}$

  - Reduced poloidal damping would make layer wider.
  - Observed layer width reduces after LH-transition due to reduced $\chi_\theta$.

# FIRST THEORIES OF H-MODE: ExB shear decorrelation

- **1990 K.C. Shaing, E.C. Crume Jr. and W.A. Houlberg, Phys. Fluids B2, 1492**
  - First published in EPS proceedings 1989 and APS-DPP 1989.
- **1990 H. Biglari, P.H. Diamond and P.W. Terry, Phys. Fluids B2, 1**
- **1992 Y.Z. Zhang and S.M. Mahajan, Phys. Fluids B4, 1385**
- **1995 T.S. Hahm and K.H. Burrell, Phys. Plasmas 2, 1648**
  - Suppression of turbulence is through reduction of the radial correlation length.
  - Toroidal form of the Hahm-Burrell shearing rate $\omega_s$

$$\left|\frac{\tilde{n}}{n}\right|^2 \cong \frac{\left|\tilde{n}/n\right|^2_{\omega_s=0}}{\left(1+\dfrac{\omega_s^2}{\Delta\omega_T^2}\right)} = \left(\frac{\Delta\psi}{\Delta\psi_0}\right)^2 \left|\tilde{n}/n\right|^2_{\omega_s=0} \qquad \omega_s = \frac{(RB_\theta)^2}{B}\frac{\partial}{\partial\psi}\left(\frac{E_R}{RB_p}\right)$$

Waltz-Miller shearing rate:
1999 Phys. Plasmas 6, 4265

$$\gamma_{ExB} = -\frac{r}{q}\frac{\partial}{\partial r}\left(\frac{c\partial\phi}{\partial\psi}\right)$$



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# FIRST THEORIES OF H-MODE: Energy transport bifurcation

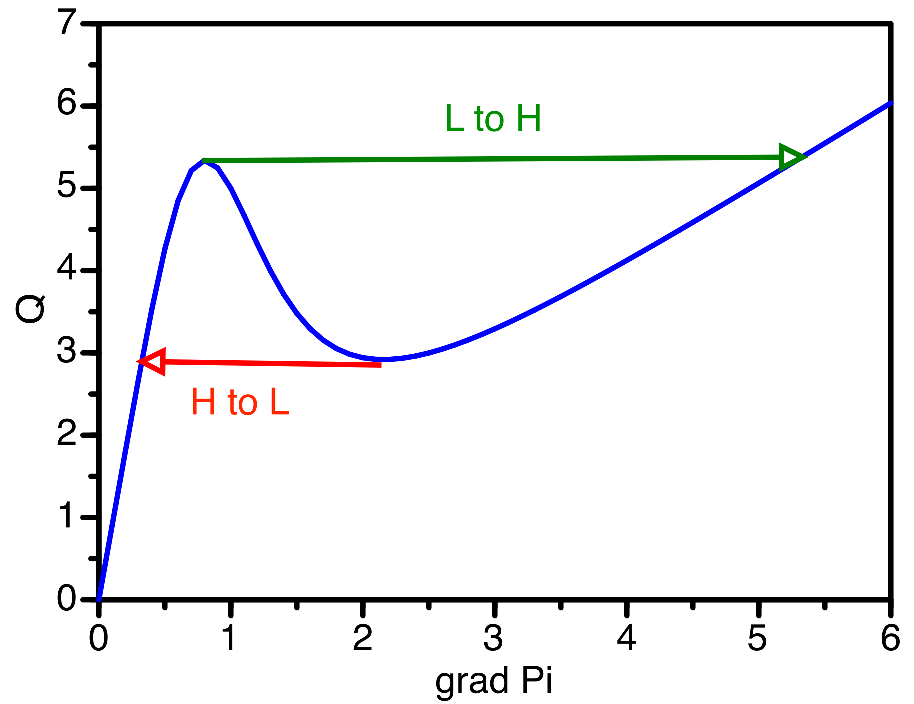
- **1993 F.L. Hinton and G.M. Staebler, Phys. Fluids B5, 1281**
  - The suppression of turbulence amplitude by ExB shear decorrelation can cause a bifurcation to improved confinement
  - The model assumes the parallel and toroidal ion velocities are zero; i.e., $V_{ExB} = -V_{dia}$ based on CER data in H-mode phase

$$\gamma_{ExB} = -\frac{\partial U_{ExB}}{\partial r} \approx \frac{\partial}{\partial r}\left(\frac{c}{eBn}\frac{\partial P_i}{\partial r}\right)$$

$$\approx -\frac{c\partial n}{eBn^2\partial r}\frac{\partial Pi}{\partial r} \approx -\frac{c}{eBn^2T_i}\left(\frac{\partial Pi}{\partial r}\right)^2$$

$$Q = -\left(\Phi^2\chi + \chi^{neo}\right)\frac{\partial P_i}{\partial r}$$

$$\Phi^2 = \frac{1}{\left(1 + c\gamma_{ExB}^2\right)} \qquad \frac{\chi}{\chi^{neo}} = 8$$



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# NUMERICAL SIMULATIONS WITH ExB SHEAR

- **Numerical simulations with ExB shear**
  - Linear stability
  - Turbulence
  - Spectral shift model
  - Momentum transport



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# NUMERICAL SIMULATIONS WITH ExB SHEAR: Linear stability

- **Gyrokinetic stability calculations in sheared slab magnetic geometry show that mean field ExB velocity shear is linearly stabilizing.**
  - Sheared slab geometry: $\vec{B} = B_0\hat{z} + \left(1+\hat{s}\left(x-x_0\right)/x_0\right)\frac{x_0 B_0}{q_0 R_0}\hat{y}$
- **ExB shear of either sign is stabilizing for both electron and ion instabilities.**
  - 1991 H. Sugama and M. Wakatani, Phys. Fluids B 3, 1110
  - 1992 X.H. Wang and P.H. Diamond, Phys. Fluids B 4, 2402
  - 1993 J.Q. Dong and W. Horton, Phys. Fluids B 5, 1581
- **ExB curvature: negative Er well of H-mode is stabilizing for ion modes, destabilizing for electron modes.**
  - 1991 G.M. Staebler and R.R. Dominguez, Nuclear Fusion, 31, 1891
- **Reynolds stress due to ExB shear Doppler term is a pinch.**
  - 1993 R.R. Dominguez and G.M. Staebler, Phys. Fluids B 5, 3876
- **In toroidal geometry linear stability is difficult. The 1D ballooning representation gives traveling waves (Floquet modes) not eigenmodes.**
  - Floquet modes do not yield a simple interpretation of the turbulence simulations.

# NUMERICAL SIMULATIONS WITH ExB SHEAR: turbulence

- **The first turbulence simulations were performed with gyro-fluid equations with adiabatic electrons.**
  - 1994 R.E. Waltz, G.D. Kerbel and J. Milovich, Phys. Plasmas 1, 2229
- **These results were later confirmed with gyrokinetic simulations**
  - 1996 A. Dimits, T.J. Williams, J.A. Byers and B.A. Cohen Phys. Rev. Lett. 77, 71.
- **It was found that equilibrium ExB velocity shear completely turned off (quenched) the turbulence if the "Waltz quench rule" was satisfied**

$$\gamma_{ExB} > Max\left[\gamma_{k_y}^{lin}\right]$$

- **The decorrelation formula of Shaing gives about a 10% reduction in the fluxes at the point where the turbulence was quenched (BDT is even weaker).**
  - 1998 R.E. Waltz, R.L. Dewar and X. Garbet, Phys. Plasmas 5, 1784.

# NUMERICAL SIMULATIONS WITH ExB SHEAR: Spectral Shift

- **The decorrelation model gives a weak intensity suppression (Shaing 1990).**

$$\bar{\bar{\Phi}}^2\left(\gamma_{ExB}\right) = \bar{\bar{\Phi}}^2(0) \Big/ \left(1 + \alpha_c\left(\gamma_{ExB}\tau_c\right)^2\right) \qquad \bar{\bar{\Phi}}^2 = \int_{-\infty}^{\infty} dk_y \int_{-\infty}^{\infty} dk_x \Phi^2_{k_x,k_y}$$

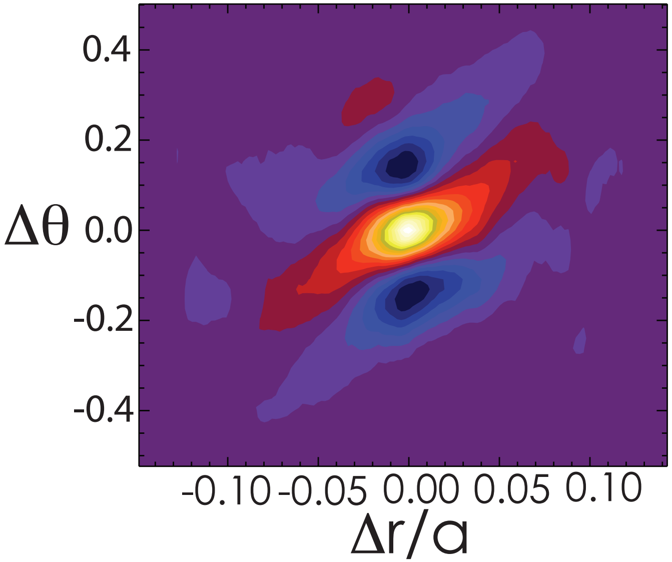
- **The quench rule gives a stronger intensity suppression (Waltz 1994).**

$$\bar{\bar{\Phi}}^2\left(\gamma_{ExB}\right) = \bar{\bar{\Phi}}^2(0)\,\mathrm{MAX}\left[1 - \alpha_E \left|\gamma_{ExB}\right| / \gamma_{max}, 0\right]$$

- **Neither formulas give a finite Reynolds stress because they do not break the parity of the eigenmodes.**

- **ExB shear breaks radial parity producing a finite spectral average $k_x$-shift and a tilt of the the 2D correlation function shown at right.**

$$\gamma_{ExB} = 0.1 c_s / a$$



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# SHEAR IN THE ExB VELOCITY DOPPLER TERM CAUSES A RADIAL WAVENUMBER SPECTRAL SHIFT

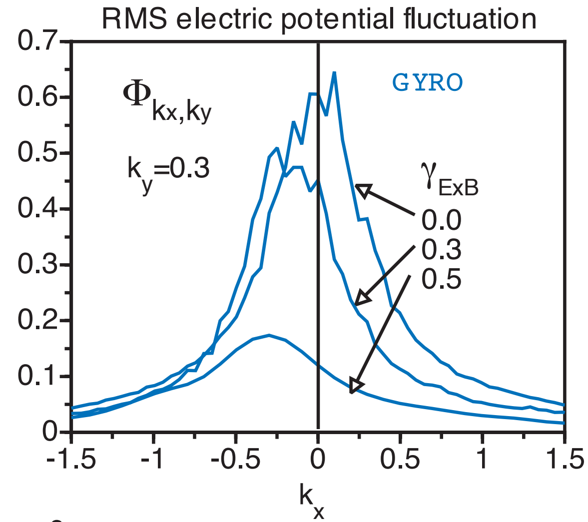
- **2013 G.M. Staebler et al., Phys. Rev. Lett. 110, 055003**
- $\Phi_{k_x,k_y}$ **The time and flux surface average RMS amplitude of the electric potential fluctuations in non-linear gyro-kinetic simulations reveals a shift in the peak of the radial wavenumber spectrum.**

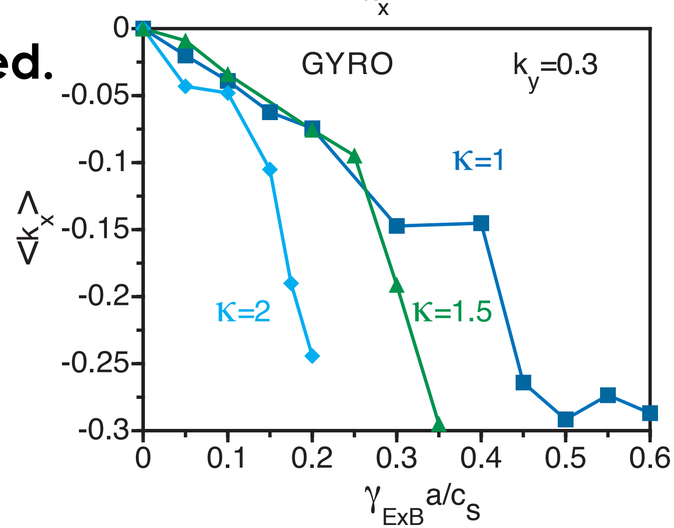
$$k_x = k_r \rho_s \qquad k_y = k_\theta \rho_s \qquad k_x = k_y \hat{s} \theta_0$$

- **The amplitude of the shifted peak is suppressed.**
- **The spectral average shift is a non-linear function of the Waltz-Miller shear rate.**

$$\langle k_x \rangle = \int_{-\infty}^{\infty} dk_x k_x \Phi^2_{k_x,k_y} \Big/ \bar{\Phi}^2_{k_y}$$

$$\bar{\Phi}^2_{k_y} = \int_{-\infty}^{\infty} dk_x \Phi^2_{k_x,k_y} \qquad \gamma_E = -\frac{q}{r}\frac{d}{dr}\left(\frac{c\partial\phi}{\partial\psi}\right)$$





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# NON-LINEAR MODEL OF THE SPECTRAL SHIFT

The RMS amplitude of the electric potential fluctuations from GYRO is well fit by a Lorentzian for $k_y > 0.05$

$$\Phi_{k_x,k_y} = \frac{\gamma_{k_y}^{eff}}{\left(c_y k_y^2 + c_x k_x^2\right)}$$

With a uniform ellipticity $\frac{c_x}{c_y} = 0.56$

The Bernoulli differential equation provides an interpretive model of the **linear growth**, **Doppler shear $k_x$-coupling** and **non-linear mixing** of the potential

$$\frac{d}{dt}\Phi_{model} = \underline{\gamma_{k_y}^{eff}\Phi_{model}} + \underline{\gamma_{ExB} k_y \frac{\partial \Phi_{model}}{\partial k_x}} - \underline{\left(c_y k_y^2 + c_x k_x^2\right)\Phi_{model}^2} = 0$$

The Doppler shear term is **linearly destabilizing** when $\gamma_{ExB} k_y \frac{\partial \Phi_{model}}{\partial k_x} > 0$
This induces a tilt of the spectrum which is then
re-centered about a shifted peak by the non-linear mixing term.


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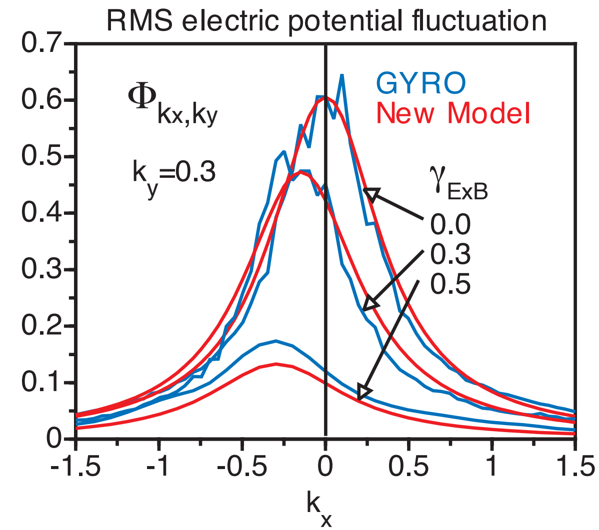
# THE SPECTRAL SHIFT MODEL FOR THE 2D-SPECTRUM

The interpretive model has the solution

$$\Phi_{\text{model}} = \frac{\gamma_{k_y}^{\text{eff}}}{\left(c_y k_y^2 + c_x \langle k_x \rangle^2 + c_x \left(k_x - \langle k_x \rangle\right)^2\right)}$$

where $\langle k_x \rangle = -k_y \gamma_{ExB} / \gamma_{k_y}^{eff}$




- The model spectrum shifts in the linearly destabilized direction with a symmetric peak.
- The peak amplitude is reduced by the reduction in the radial correlation similar to the decorrelation formula of Hahm: $\Delta_r^2/\rho_s^2 \approx \langle k_x^2 \rangle^{-1} = c_x / \left(c_y k_y^2 + 2c_x \langle k_x \rangle^2\right)$
- **This reduction is far too weak compared to GYRO.**

The ansatz $\gamma_{k_y}^{\text{eff}}\left(\langle k_x \rangle\right) = \gamma_{k_y}^{\text{eff}}(0) / \left(1 + \left(\alpha_x \langle k_x \rangle / k_y\right)^4\right)$ with $\alpha_x = 1.15$, $c_x = 0.56 c_y$
**robustly fits the GYRO simulations**
independent of plasma parameters (q,r/R,s,Ti/Te,elongation).
This completes the "**spectral shift**" model.



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# TRAVELING WAVES ARE LOCALIZED BY NONLINEAR MIXING

- **The linear modes of the convective derivative are traveling waves:**

$\gamma^{\mathrm{eff}}_{k_y} \rightarrow \frac{\partial}{\partial t}$ Gives the linear equation

$$\frac{d}{dt}\Phi_{\mathrm{model}} = \frac{\partial}{\partial t}\Phi_{\mathrm{model}} + \gamma_{\mathrm{ExB}} k_y \frac{\partial \Phi_{\mathrm{model}}}{\partial k_x} = 0$$

Which has traveling wave solutions: Floquet modes

$$\Phi_{\mathrm{model}} = \Phi_{\mathrm{model}}\left(k_y, k_x - k_y \gamma_{\mathrm{ExB}} t\right)$$

The non-linear term localizes the traveling waves
To standing waves of the form

$$\Phi_{\mathrm{model}} = \Phi_{\mathrm{model}}\left(k_y, k_x + k_y\, \gamma_{\mathrm{ExB}} / \gamma^{\mathrm{eff}}_{k_y}\right) = \Phi_{\mathrm{model}}\left(k_y, k_x - \langle k_x \rangle\right)$$


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# THE SPECTRAL SHIFT MODEL APPLIED TO TGLF

- **The quasi-linear Trapped Gyro-Landau Fluid (TGLF) transport model computes linear drift-wave ballooning eigenmodes and quasi-linear weights at the peak of the shifted spectrum.**
- **A formula for the spectral shift has been fit to GYRO simulations.**
- 2013 G.M. Staebler et al., Nuclear Fusion 53, 113017

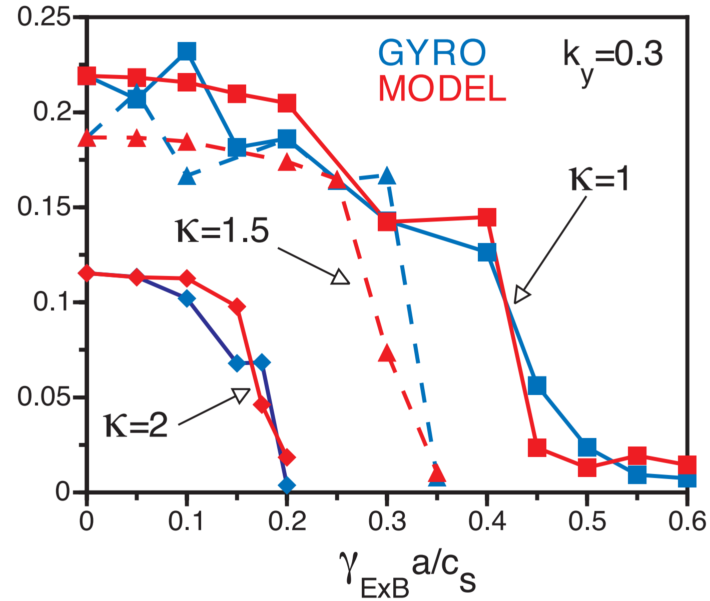
$$\Phi\left(\langle k_x \rangle, k_y\right)$$

$$\langle k_x \rangle_{fit} = k_y \mathrm{MIN}\left[k_{x_e}/k_y, 1.3\right] \quad \text{where}$$

$$k_{x_e} = -k_y \left(\frac{\gamma_{ExB}}{\gamma_{0,k_y}}\right)\left\{0.36 + 0.38\sigma_x \mathrm{Tanh}\left[\left(0.69\sigma_x \frac{\gamma_{ExB}}{\gamma_{0,k_y}}\right)^6\right]\right\}$$

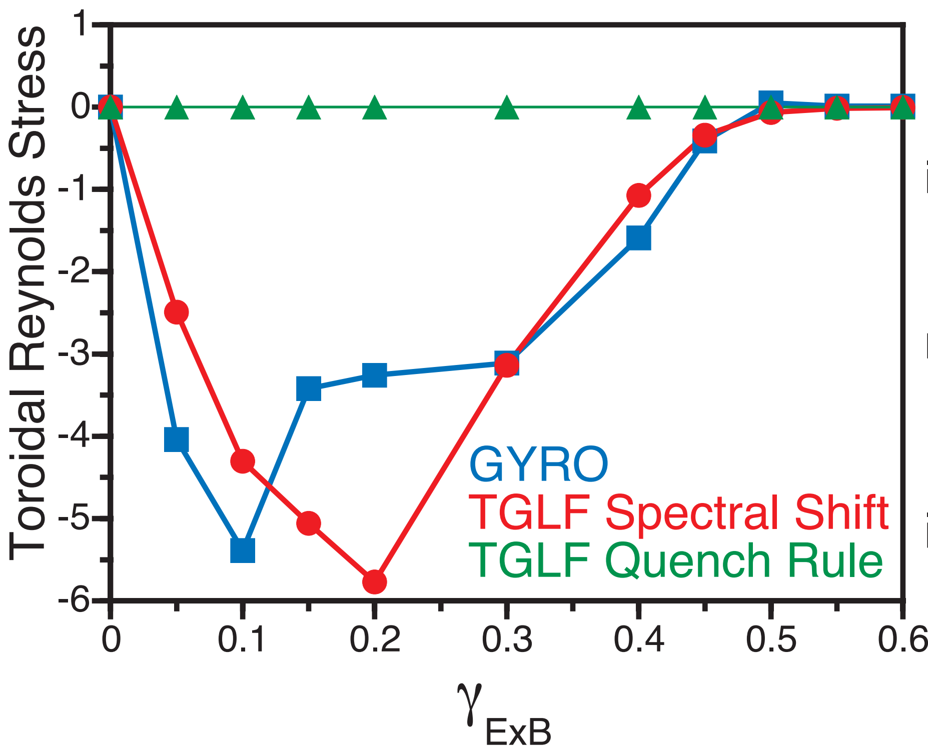
$$\sigma_x = \mathrm{MIN}\left[k_y/0.3, 1.0\right]\left(1 + 0.42\left(1 - B_{unit}/B(0)\right)^2\right)$$

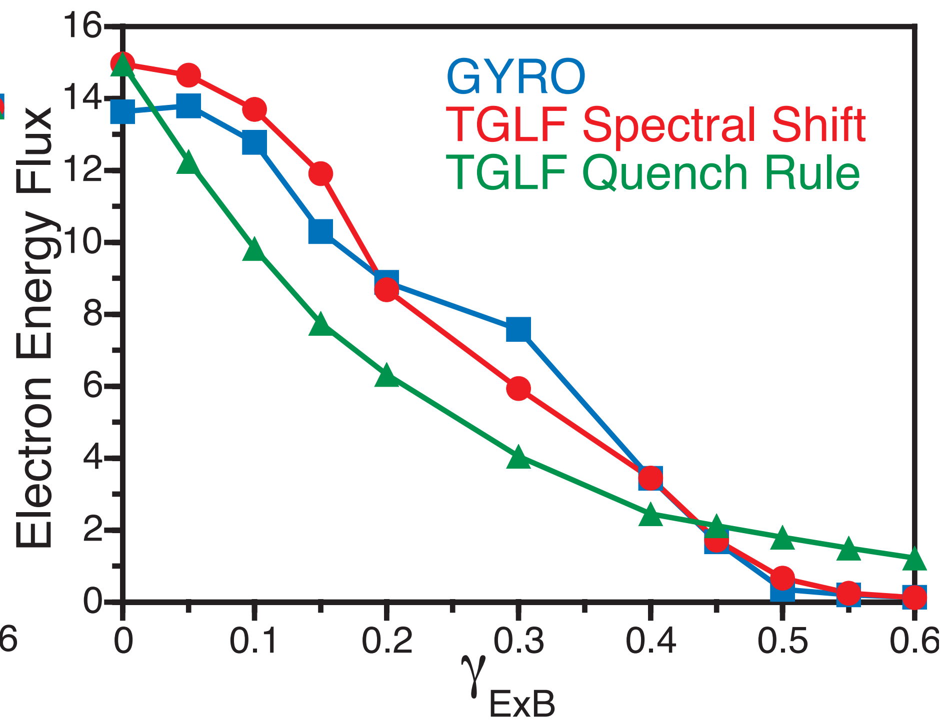
where $\gamma_{0,k_y}$ = linear growthrate at $k_x = 0, k_y$



# THE SPECTRAL SHIFT MODEL ALLOWS QUASI-LINEAR CALCULATION OF THE REYNOLDS STRESS MOMENTUM PINCH

- The Reynolds stress is due to the finite $k_x$ of the linear modes.
- The accuracy of TGLF in modeling the GYRO energy transport is improved with the spectral shift compared to the quench rule.



# NUMERICAL SIMULATIONS WITH ExB SHEAR: Momentum

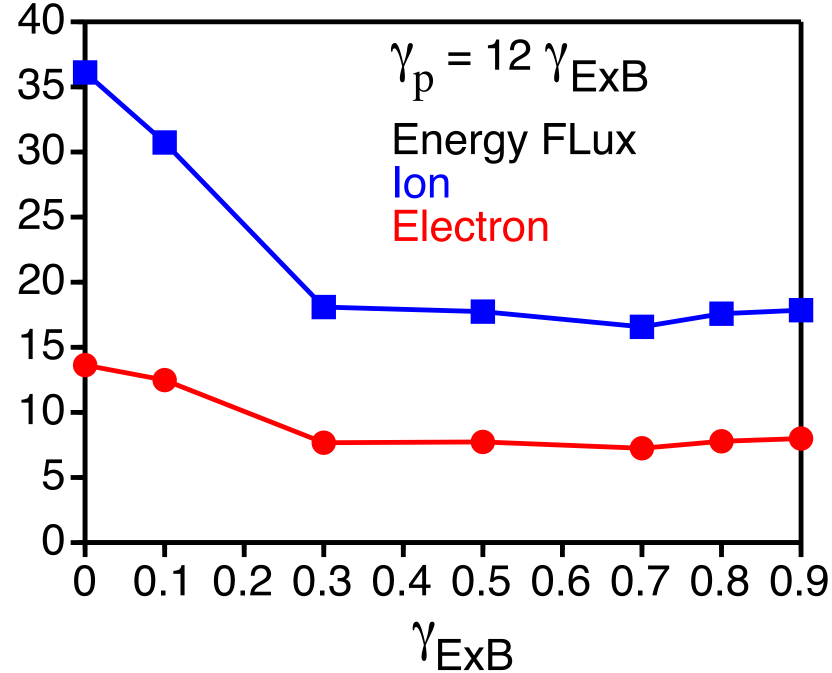
- **The properties of the equilibrium which produce a Reynolds stress have been explored with gyrokinetic turbulence simulations.**
- **Parallel velocity shear - Kelvin Helmholtz mode**
- **Parallel velocity – Corriolis drift**
- **Up/Down flux surface asymmetry**
- **ExB velocity shear – shear in the Doppler shift**
- **Profile shear – radial variation in the drift frequencies**
- **All of these except profile shear are included in the TGLF quasilinear transport model.**
  - 2011 G.M. Staebler, R.E. Waltz and J.E. Kinsey, Phys. Plasmas 18, 056106

# PARALLEL VELOCITY SHEAR REDUCES THE TRANSPORT SUPPRESSION DUE TO ExB VELOCITY SHEAR

- **Parallel velocity shear drives a Kelvin-Helmholtz (KH) type instability.**
- **The increase in the linear growth rate at large parallel velocity shear arrests the reduction in energy transport due to ExB velocity shear from the Doppler shift.**
  - This was observed in gyrofluid (Waltz 1998) and gyrokinetic (Kinsey 2005, Highcock 2010) turbulence simulations.

$$\gamma_p = R\frac{d}{dr}\left(\frac{c\partial\phi}{\partial\psi}\right) = \frac{Rq}{r}\gamma_{ExB}$$

**GYRO turbulence simulations**



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# MEAN FIELDS AND FLUCTUATIONS

- **Mean fields and fluctuations**
  - Separation between driftwave turbulence and transport space and time scales
  - Zonal fluctuation mediated saturation and the Dimits shift

# MEAN FIELDS AND FLUCTUATIONS

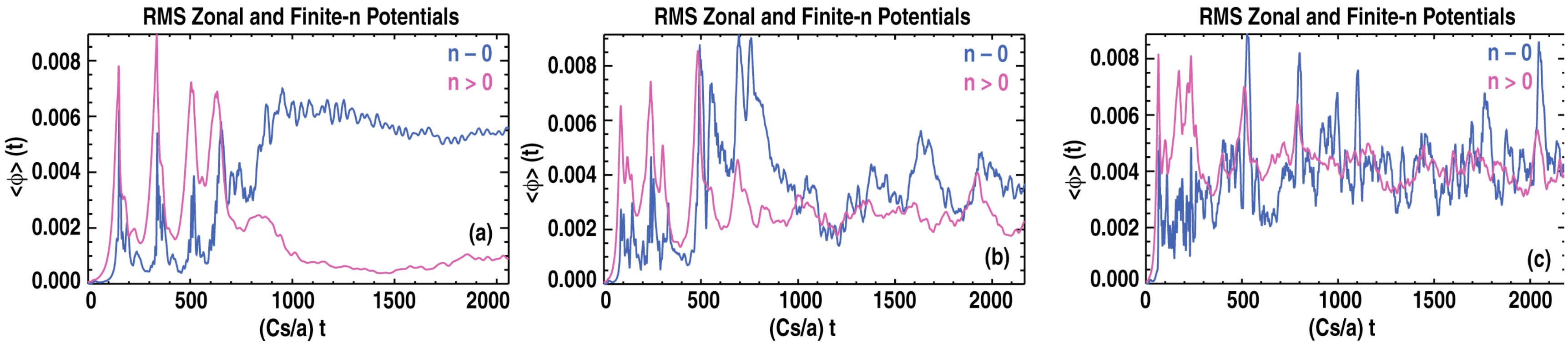
- **The starting point for delta-f gyrokinetic theory of turbulence is to use an ensemble average to separate the fluctuations from the mean fields.** $f = F + \tilde{f}$ $\quad F = \{f\}$ $\quad \{\tilde{f}\} = 0$
  - 1982 E.A. Frieman and L. Chen, Phys. Fluids 25, 502.
  - The ensemble average { * } includes a time average and a radial average over a flux tube.
  - The time average should be longer than the decorrelation time of the turbulence ~ 1/drift frequency ~ $R/\sqrt{T_i/m_i}$ ~ $1-10\mu s$
  - The radial flux tube width (mean field gradient length) should be longer than the radial correlation length of the turbulence.
  - The correlation length is typically 5-10 $\rho_s$ ~ 0.2 – 0.4 cm for DIII-D.
    - 2000 T.L. Rhodes et al., Phys. Plasmas 9, 2141.

# MEAN FIELDS AND FLUCTUATIONS: Zonal flows

- **The axisymmetric ExB "zonal flows" can be separated into mean field and fluctuating contributions.** $v_{ExB} = \left\{ v_{ExB} \right\} + \tilde{v}_{ExB}$ $\left\{ \tilde{v}_{ExB} \right\} = 0$
  - The fluctuating zonal flows are part of the gyrokinetic equations (1 $\mu$s).
  - The mean ExB velocity is evolved by the transport equations (100 $\mu$s).
- **The zonal ExB fluctuations in flux tube gyrokinetic simulations always have a symmetric spectrum of radial wavenumbers ($k_x$) even when the finite poloidal wavenumber $k_y$ spectrum is shifted in $k_x$ by mean field flows.**
  - There is no spectral shift (eddy tilting) induced by zonal fluctuations.
  - There is no momentum transport induced by zonal fluctuations.
  - Zonal fluctuations are linearly stable but are driven by non-linear ExB advection of the finite $k_y$ fluctuations.
  - Zonal fluctuations have a broadband frequency spectrum and resonant frequency geodesic acoustic modes (GAMS).
  - Zonal fluctuations have no net perpendicular ion velocity fluctuation.
    – Radial force balance between ExB and ion diamagnetic velocities at sub-correlation length scales.

# THE ZONAL ELECTRIC POTENTIAL FLUCTUATIONS BECOMES DOMINANT IN THE DIMITS SHIFT REGIME

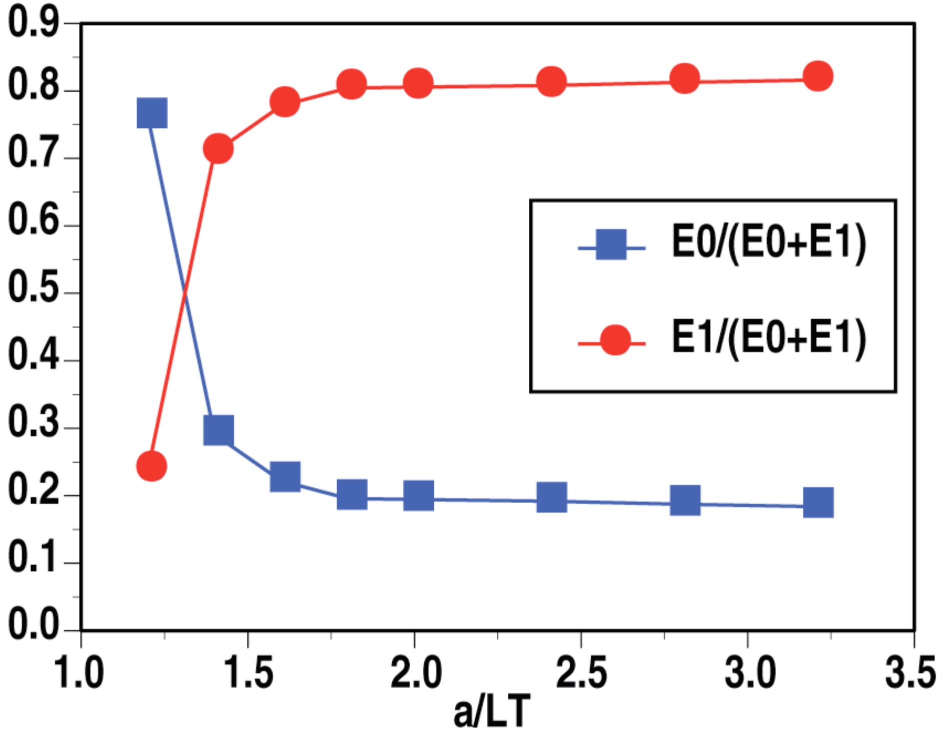
- **The fluctuating zonal (n=0) electric potential RMS amplitude can become large compared to the sum of the n>0 potential RMS amplitudes.**
- **The plasma parameters impact the range of existence of the Dimits shift regime. (weak magnetic shear, low-q, peaked density, …)**
  - 1996 A. Dimits, T.J. Williams, J.A. Byers and B.A. Cohen Phys. Rev. Lett. 77, 71.




RMS zonal (n=0) and finite-n (n>0) electric potential fluctuation amplitudes for a/LT=1.2 (a), 1.4 (b) and 1.6 (c) for kinetic electron GYRO simulations

# THE DIMITS SHIFT REGIME ONLY EXIST NEAR THE LINEAR STABILITY POINT

- **The zonal electric field energy $E_0$ is larger than the finite-n electric field energy $E_1$ in the Dimits shift regime**



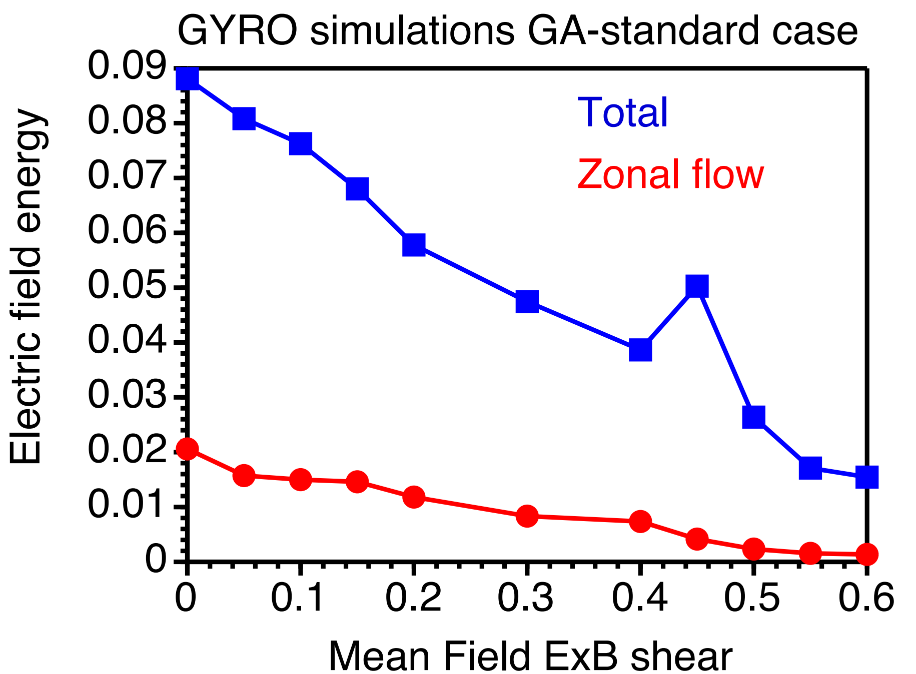

$$E_1 = \sum_{n>0} \left|\nabla \tilde{\Phi}_n\right|^2$$

$$E_0 = \left|\nabla \tilde{\Phi}_0\right|^2$$

# IT IS UNLIKELY THAT THE H-MODE IS TRIGGERED BY A STATE OF ZONAL FLUCTUATION IMBALANCE

- **There are known conditions where zonal flows are large compared to the finite-n turbulence.**
  - The Dimits shift near the critical gradient where there are only a few finite-n modes unstable.
  - Near low-order rational q-surfaces with weak magnetic shear.

  2006 M. Austin et al., Phys. Plasma 13, 082502
- **Increasing the temperature gradients in delta-f gyrokinetic simulations has <u>never</u> been found to cause a reduction of the transport due to an imbalance of zonal flows.**
- **H-mode is not a state of high zonal fluctuations like the Dimits shift.**




- **As the mean field ExB shear is increased, the zonal flow fractional contribution to the total fluctuating electric field energy is reduced from 23% to 9%**

# PREDATOR-PREY TURBULENCE SATURATION MODEL

- **1994 P.H. Diamond, Y.-M. Liang, B.A. Carreras and P.W. Terry, Phys. Rev. Lett. 72, 2565**
  - This paper interpreted the zonal flow dominant regime as an H-mode.
  - 2001 M. Malkov, P.H. Diamond, M. Rosenbluth, Phys. Plasmas 8, 5073
  - 2003 Eun-jin Kim and P.H. Diamond, Phys. Rev. Lett. 90, 185006
  - 2005 P.H. Diamond, et al., Plasma Phys. Contr. Fusion 47, R35
  - Heuristic dynamical equations for the turbulence intensity E and zonal fluctuation shear $V_{ZF}$.
  - This model has the form of a Lotka-Volterra "predator-prey" model with an extra self-damping term $a_1$ (the prey are cannibals!)

$$E = \Phi^2$$

$$V_{ZF} = \left|\frac{\partial \tilde{u}_{ExB}}{\partial r}\right|$$

$$\partial_t E = \gamma_0 E - a_3 V_{ZF}^2 E - a_1 E^2$$

$$\partial_t \mathrm{V}_{\mathrm{ZF}} = \mathrm{b}_1 \mathrm{E} \mathrm{V}_{\mathrm{ZF}} - \mathrm{b}_3 \mathrm{V}_{\mathrm{ZF}}$$



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# THE MODEL DAMPS TO A STEADY STATE AFTER TRANSIENT OSCILLATIONS

- **The qualitative features of the Dimits shift and normal regimes can be modeled for different values of the zonal flow damping rate** $b_3$
- Without the self-damping term the oscillations continue undamped.

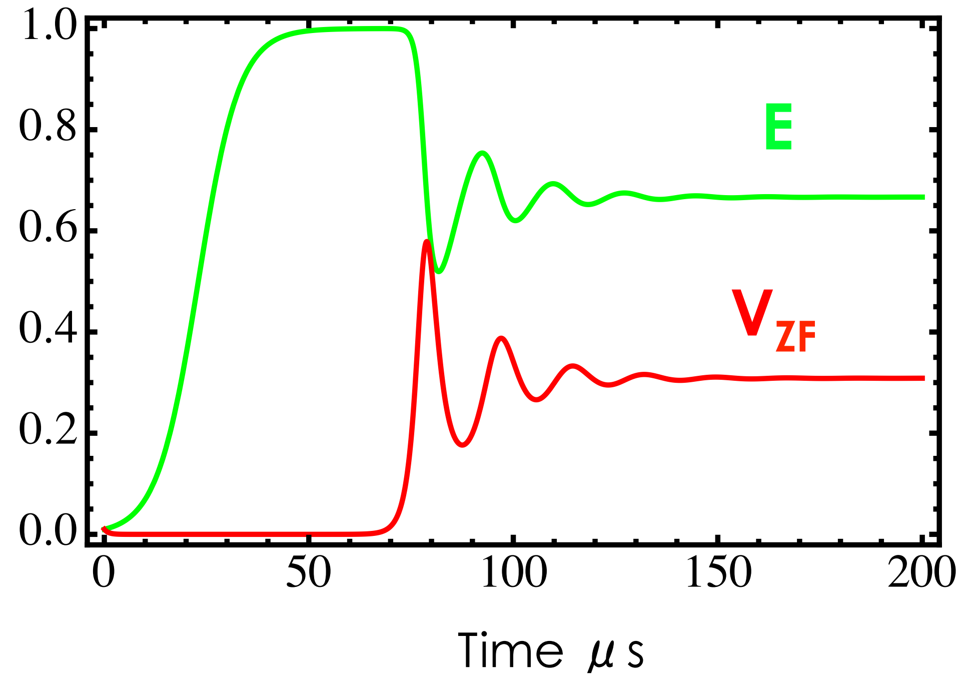
The E = 0, U = 0 equilibrium is linearly unstable for $\gamma_0 > 0$
The system rapidly evolves to a unique stationary state

$$E = \frac{b_3}{b_1} \qquad V_{ZF}^2 = \frac{\gamma_0 - a_1 \frac{b_3}{b_1}}{a_3}$$

**The time scale is set by the linear growthrate**

$$\tau = 1/\gamma_0 = 5\mu s$$

**P-P oscillation frequency ~ 50kHz**




$$a_1 = 0.2,\ a_3 = 0.7,\ b_1 = 1.5,\ b_3 = 1,\ \gamma_0 = 0.2$$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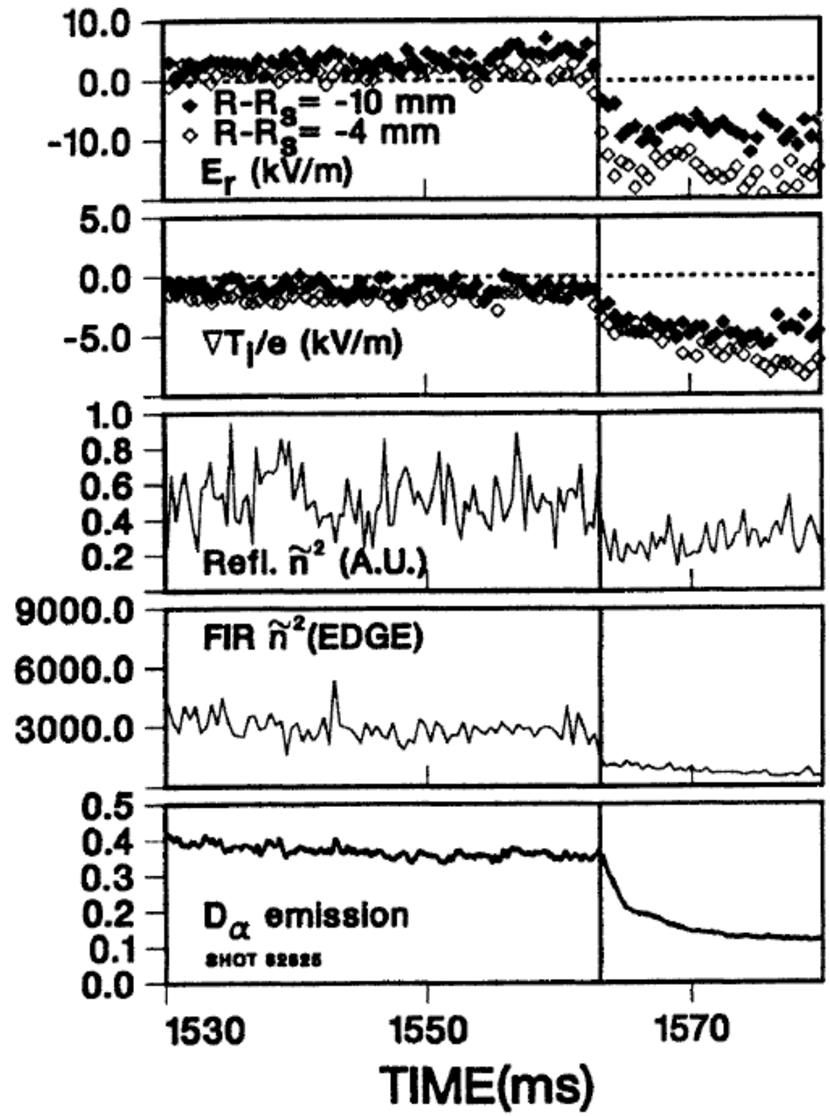
# TRIGGERS & DITHERS

- **Triggers & Dithers**
  - CER data finds a trigger event
  - DBS movies of dithering transitions
  - Kim-Diamond model of dithering LH-transitions
  - Mean field transport limit cycle oscillations

# TRIGGERS & DITHERS: CER finds a trigger event

- **The CER measures velocities with an integration time of 500μs .**
  - **These are mean fields.**
- **The L/H transition has two time scales.**
  - The ExB velocity and density fluctuation amplitude (Refl. 100 $\mu$ s) change on a fast time scale (trigger). 1991 E.J. Doyle, Phys. Fluids B 3, 2300.
  - The diamagnetic velocity is observed to change more slowly (lock-in H-mode).
- **The CER measured carbon poloidal velocity balances the ExB velocity trigger – not toroidal velocity**
  - 1996 K.H. Burrell PPCF 38, 1313



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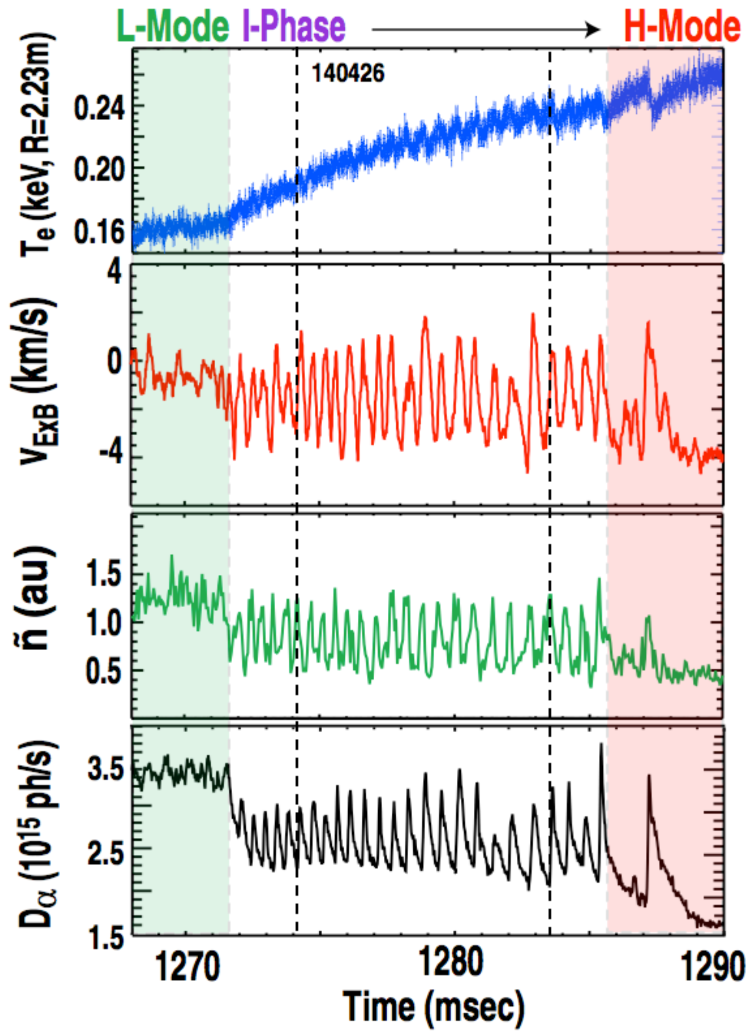
# TRIGGERS & DITHERS: DBS movies of dithering transitions

- **Doppler backscattering (DBS) data measures the phase velocity of the turbulence on a fast timescale.**
  2012 L. Schmitz et al., Phys. Rev. Lett. 108, 155002
- **DBS data averaged over 100 $\mu$ s >> decorrelation time of 2 $\mu$ s**
  - **These are mean fields.**
- **The ExB velocity and density fluctuation amplitude are observed to change faster than the temperature profiles at each dither.**
  - Similar to the L/H "trigger" but repeated at each cycle

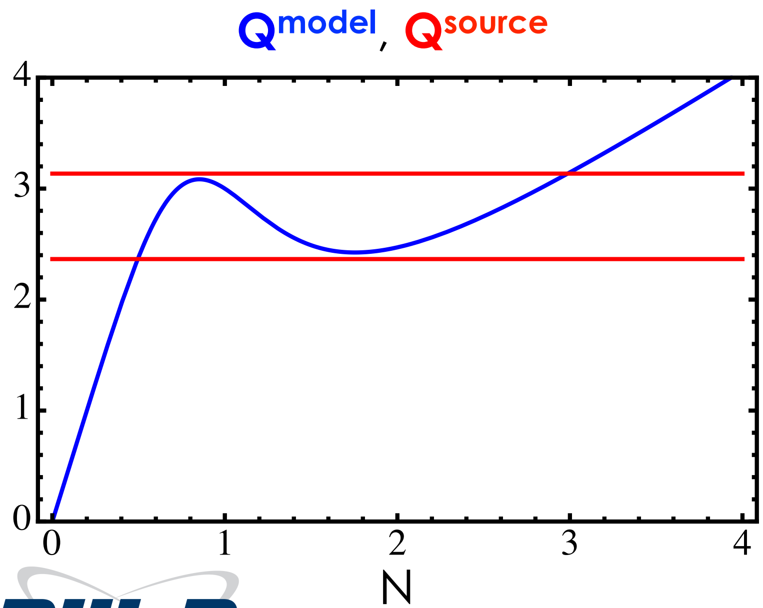


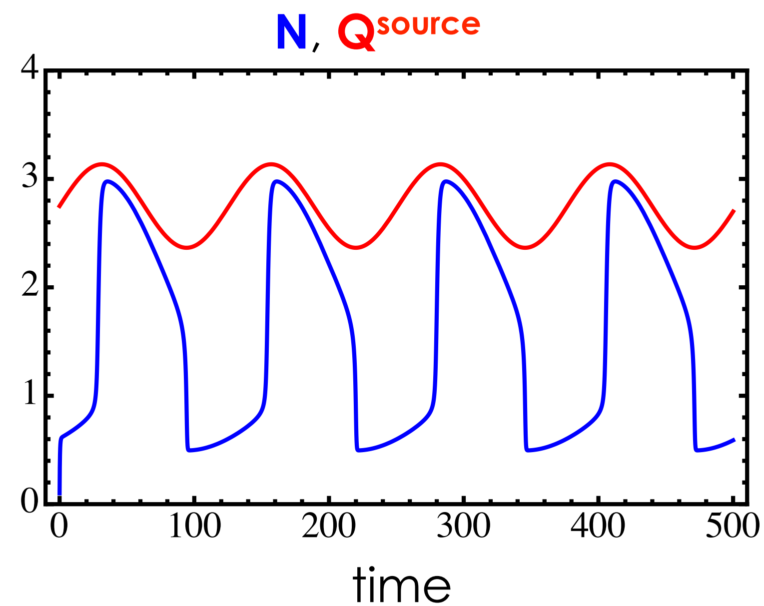
# PERIODIC FORCING CAN CAUSE LIMIT CYCLE OSCILLATIONS BETWEEN L AND H STATES

- **If the flow of heat across a flux surface is intermittent in the range of hysteresis it can cause limit cycle oscillations.**
  - This is illustrated with the heat flux bifurcation of the Hinton-Staebler model. N = mean field pressure gradient

$$\partial_t N = Q^{\text{model}} + Q^{\text{source}}(t) \qquad Q^{\text{model}} = \left(\frac{d_1}{1+V^2} + d_2\right)N \qquad V = N^2$$

$$d_1 = 1.0,\ d_2 = 4.0,\ Q = 2.75\left(1 + 0.14 Sin[0.05t]\right)$$



# OTHER LH-TRANSITION MODELS CAN HAVE LCOs

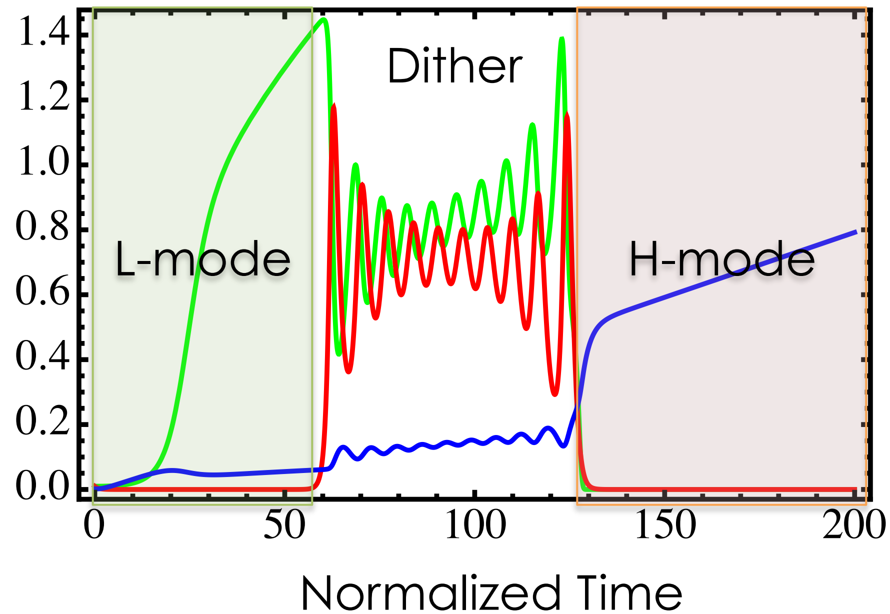
- **Limit cycle oscillations occur between the L and H stable equilibria over a range of parameters if the system is kept in the unstable middle zone.**
- **1991 S.I. Itoh et al., Phys. Rev. Lett. 67, 2485**
- **1996 V. Rhozhansky et al., Plasma Phys. Control. Fusion 38, 1327**

# COMMENTS ON THE KIM-DIAMOND MODEL

- **The Kim-Diamond model couples the damped predator-prey model to a Hinton-Staebler type energy transport bifurcation model.**

2003 Eun-jin Kim and P.H. Diamond, Phys. Rev. Lett. 90, 185006

- **Problems with the K-D model:**
- **The L-mode is a state of high turbulence and high zonal flows.**
- **The oscillations between zonal flows and turbulence are physically at the turbulence decorrelation rate ~ 500 kHZ >> dither frequency ~ 1KHz.**
- **The equations used for the mean field ExB velocity constrain the parallel and toroidal ion velocity to zero.** i.e. $V_{ExB} = -V_{dia}$




**Density fluctuation intensity**
**Zonal flow velocity**
**Pressure gradient/5**
**The heat source is ramped up in time slower ramp -> oscillations damp away before the transition to H-mode.**

# TRIGGERS & DITHERS

- **Triggers & Dithers**
  - CER data finds a trigger event
  - DBS movies of dithering transitions
  - Kim-Diamond model of dithering LH-transitions
  - **Mean field transport limit cycle oscillations**

# THERE ARE THREE MEAN FIELD VELOCITY FLUX FUNCTIONS TO BE DETERMINED BY THE TRANSPORT EQUATIONS

- **The mean field transport equations including contributions from both Coulomb collisions and turbulence have been derived using traditional time scale orderings.** (Shaing 1988, Sugama & Horton 1997)
- **The 1st order in $r_s/L$ mean field velocity is within a flux surface and incompressible.**

$$\vec{u}_a = u_{a,\parallel}\frac{\vec{B}}{B} - \frac{c\vec{B}}{B^2}\times\left(\vec{E} - \frac{\vec{\nabla}P_a}{e_a n_a}\right) = \frac{\vec{B}}{B_{ref}}u_{a,pol}(r) + \frac{R^2\vec{\nabla}\varphi}{R_{ref}}\left[u_{ExB}(r) + u_{a,dia}(r)\right]$$

- **Both the total toroidal and parallel momentum transport equations have no explicit collision terms due to momentum conservation.**

$$\sum_a\left[\frac{\partial}{\partial t}\left\langle m_a n_a R^2\vec{\nabla}\varphi\cdot\vec{u}_a\right\rangle + \left\langle R^2\vec{\nabla}\varphi\cdot\left(\vec{\nabla}\cdot\vec{\vec{\Pi}}_a\right)\right\rangle\right] = \sum_a\left\langle R^2\vec{\nabla}\varphi\cdot\vec{S}_a\right\rangle$$

$$\sum_a\left[\frac{\partial}{\partial t}\left\langle m_a n_a \vec{B}\cdot\vec{u}_a\right\rangle + \left\langle \vec{B}\cdot\left(\vec{\nabla}\cdot\vec{\vec{\Pi}}_a\right)\right\rangle\right] = \sum_a\left\langle \vec{B}\cdot\vec{S}_a\right\rangle$$

- **Neoclassical theory determines the electron poloidal velocity**

# A REDUCED MODEL OF THE TRANSPORT EQUATIONS

2015 G.M. Staebler and R.J. Groebner, Nuclear Fusion 55, 073008

$$\frac{\partial}{\partial t}\mathrm{n} + \frac{\partial \Gamma}{\partial r} = S_n$$

**Electron density**

$$\frac{3}{2}\frac{\partial}{\partial t}\mathrm{nT}_e + \frac{\partial \mathrm{Q}_e}{\partial r} + \mathrm{n}\nu_{\mathrm{exch}}\left(\mathrm{T}_e\text{-}\mathrm{T}_i\right) = S_{T_e}$$

**Electron temperature**

$$\frac{3}{2}\frac{\partial}{\partial t}\mathrm{nT}_i + \frac{\partial \mathrm{Q}_i}{\partial r} + \mathrm{n}\nu_{\mathrm{exch}}\left(\mathrm{T}_i\text{-}\mathrm{T}_e\right) = S_{T_i}$$

**Ion temperature**

$$\mathrm{m}\frac{\partial}{\partial \mathrm{t}}\mathrm{nu}_{\mathrm{tor}} + \frac{\partial \Pi_{\mathrm{tor}}}{\partial r} = \mathrm{S}_{\mathrm{tor}}$$

**Ion toroidal momentum**

$$\mathrm{mc}_{\mathrm{per}}\frac{\partial}{\partial \mathrm{t}}\mathrm{nu}_{\mathrm{par}} + \frac{\partial \Pi_{\mathrm{par}}}{\partial r} + \boxed{\mathrm{mnc}_{\mathrm{par}}\nu_{\mathrm{pol}}\left(\mathrm{u}_{\mathrm{pol}} - \mathrm{u}_{\mathrm{pol}}^{\mathrm{neo}}\right)} = \mathrm{S}_{\mathrm{par}}$$

**Ion parallel momentum**

# TURBULENCE DRIVEN AND NEOCLASSICAL TRANSPORT ARE MODELED

$$\Gamma = -\Phi^2 D^{\mathrm{lowk}}\left(\frac{\partial n}{\partial r} - \boxed{0.12\frac{n}{T_e}\frac{\partial T_e}{\partial r}}\right) - D^{\mathrm{neo}}\frac{\partial n}{\partial r}$$

$$Q_e = -n\left(\Phi^2\chi_e^{\mathrm{lowk}} \boxed{+\chi_e^{\mathrm{highk}}} + \chi_e^{\mathrm{neo}}\right)\frac{\partial T_e}{\partial r}$$

$$Q_i = -n\left(\Phi^2\chi_i^{\mathrm{lowk}} + \chi_i^{\mathrm{neo}}\right)\frac{\partial T_i}{\partial r}$$

$$\Pi_{\mathrm{tor}} = \frac{c_{\mathrm{per}}^2}{c_{\mathrm{par}}} m n_0 \boxed{\left\{-\Phi^2 d_{\mathrm{tor}}^{lowk}\frac{\partial}{\partial r}\left(u_{\mathrm{par}} - \beta_{\mathrm{tor}} u_{\mathrm{ExB}}\right)\right\}} + mn\left\{-d_{\mathrm{neo}}\frac{\partial u_{\mathrm{tor}}}{\partial r}\right\}$$

$$\Pi_{\mathrm{par}} = c_{\mathrm{per}} m n_0 \boxed{\left\{-\Phi^2 d_{\mathrm{par}}^{\mathrm{lowk}}\frac{\partial}{\partial r}\left(u_{\mathrm{par}} - \beta_{\mathrm{par}} u_{\mathrm{ExB}}\right)\right\}} + c_{\mathrm{per}} mn\left\{-d_{\mathrm{neo}}\frac{\partial u_{\mathrm{par}}}{\partial r}\right\}$$

The spectral shift model is used for $\Phi^2$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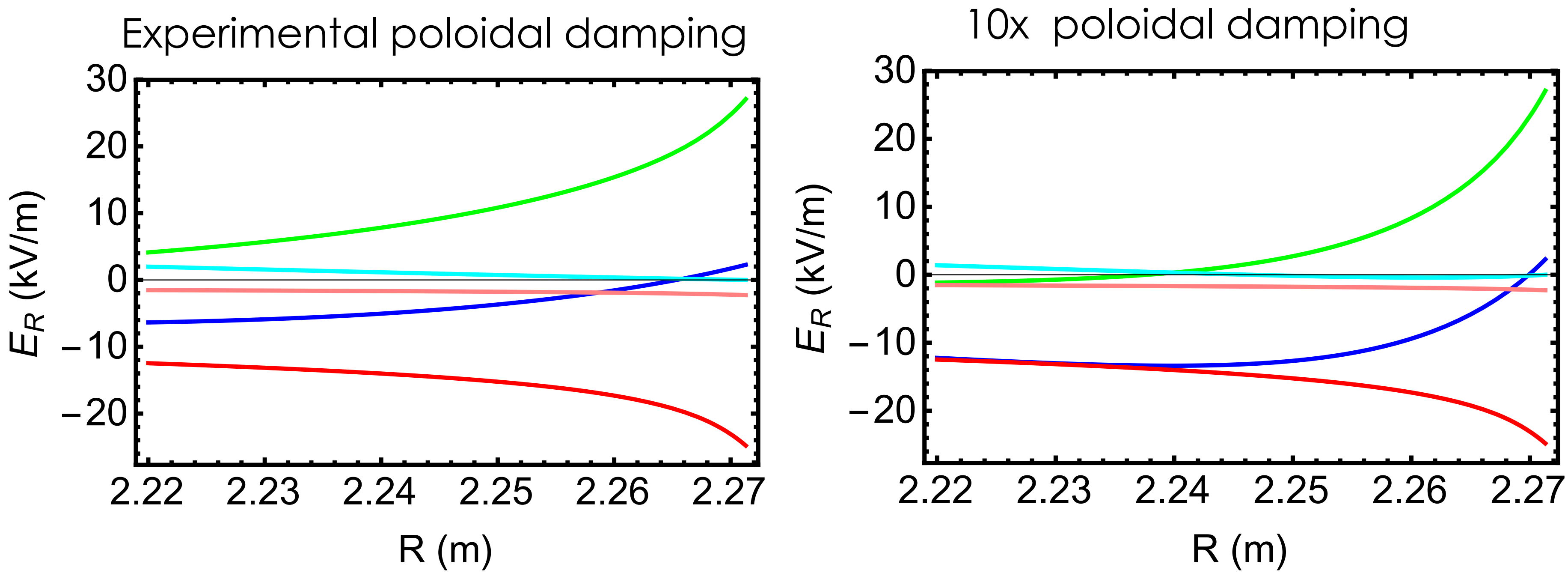
# THE VISCOUS SHEAR LAYER DETERMINES THE PROFILE OF THE RADIAL ELECTRIC FIELD

$$E_R = (R\,B_z/R_{ref})(U_{ExB})$$
$$= (R\,B_z/R_{ref})\,(U_{par} - U_{pol}\,c_{par}/c_{per} - U_{dia})$$

**Separatrix boundary conditions:** $E_R = -2.8/e\ dTe/dR$ , $U_{par} = 0.0$

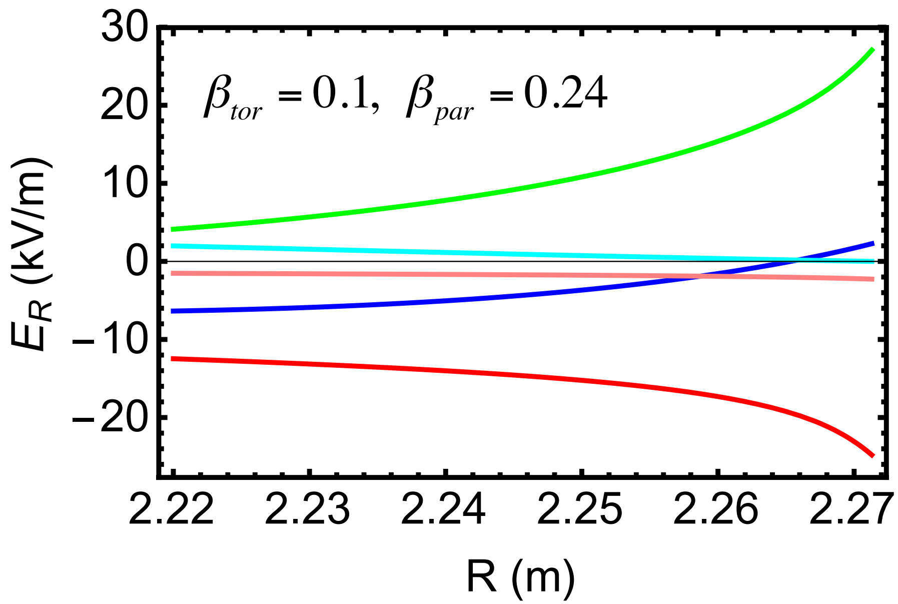


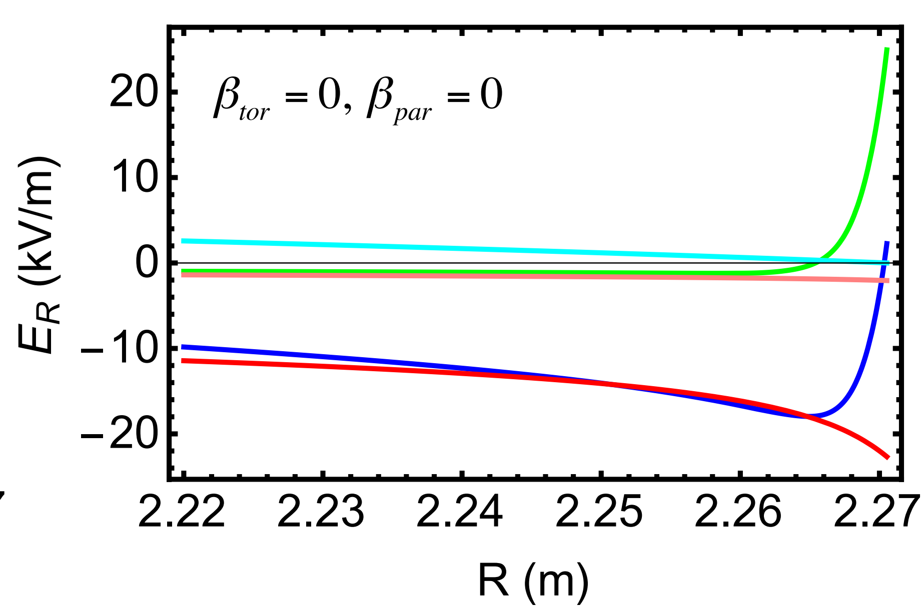
# THE DOPPLER SHEAR PINCH EFFECT IS CRITICAL

- **Without the Doppler shear pinch the parallel and toroidal stresses due to turbulence are degenerate and neoclassical momentum transport sets the very narrow width of the viscous shear layer.**

$$\Pi_{tor} = \frac{c_{per}^2}{c_{par}} mn\left\{-\Phi^2 d_{tor}^{lowk}\frac{\partial}{\partial r}\left(u_{par} - \beta_{tor} u_{ExB}\right)\right\} + mn\left\{-d_{neo}\frac{\partial u_{tor}}{\partial r}\right\}$$

$$\Pi_{par} = c_{per} mn\left\{-\Phi^2 d_{par}^{lowk}\frac{\partial}{\partial r}\left(u_{par} - \beta_{par} u_{ExB}\right)\right\} + c_{per} mn\left\{-d_{neo}\frac{\partial u_{par}}{\partial r}\right\}$$

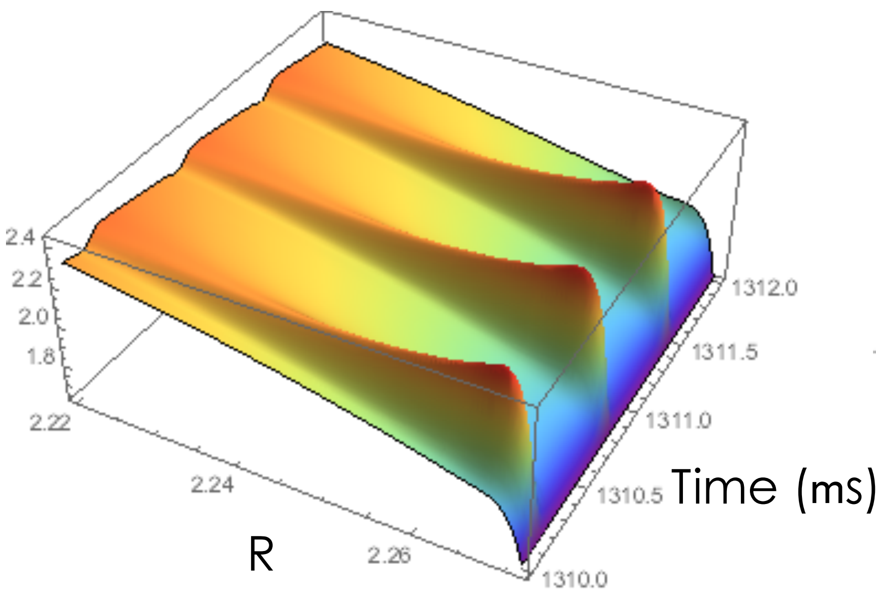


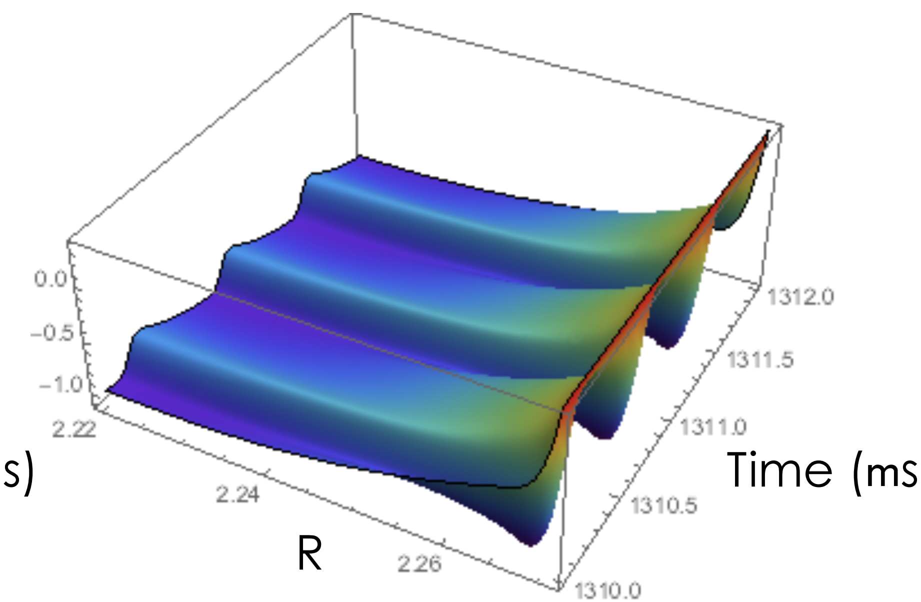
# LIMIT CYCLE OSCILLATION SOLUTIONS TO THE TRANSPORT EQUATIONS CAN MATCH EXPERIMENTAL OBSERVATIONS

- **The reduced model, constrained by data, can match the frequency and duration of the dithering for a choice of** $\left(\beta_{tor}, \beta_{par}, \gamma\right)$
- **A large transport reduction factor of $a_r$= 8 is used for this case**

Density ($10^{13}/cm^3$)




Toroidal ExB velocity ($10^5$m/s)





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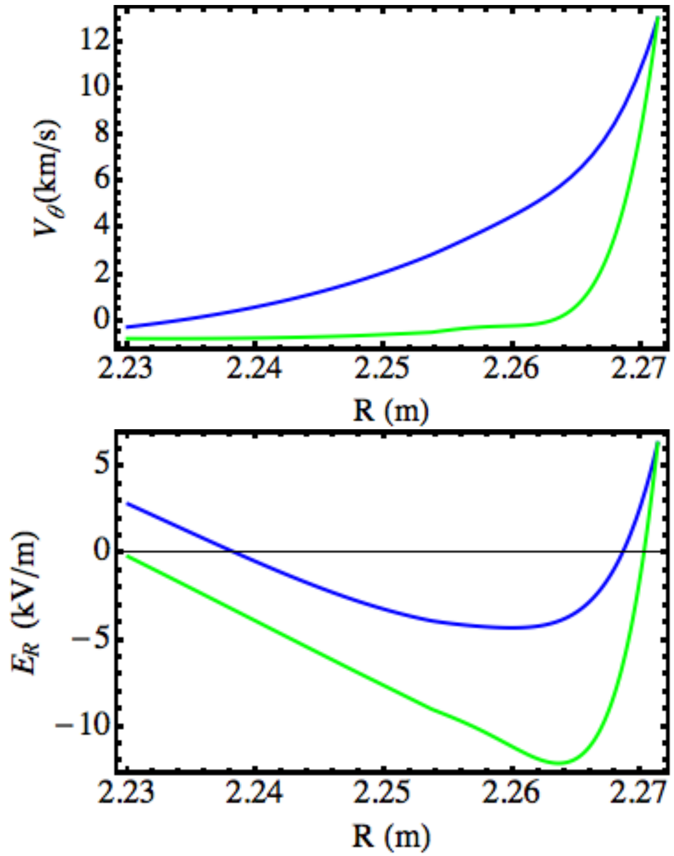
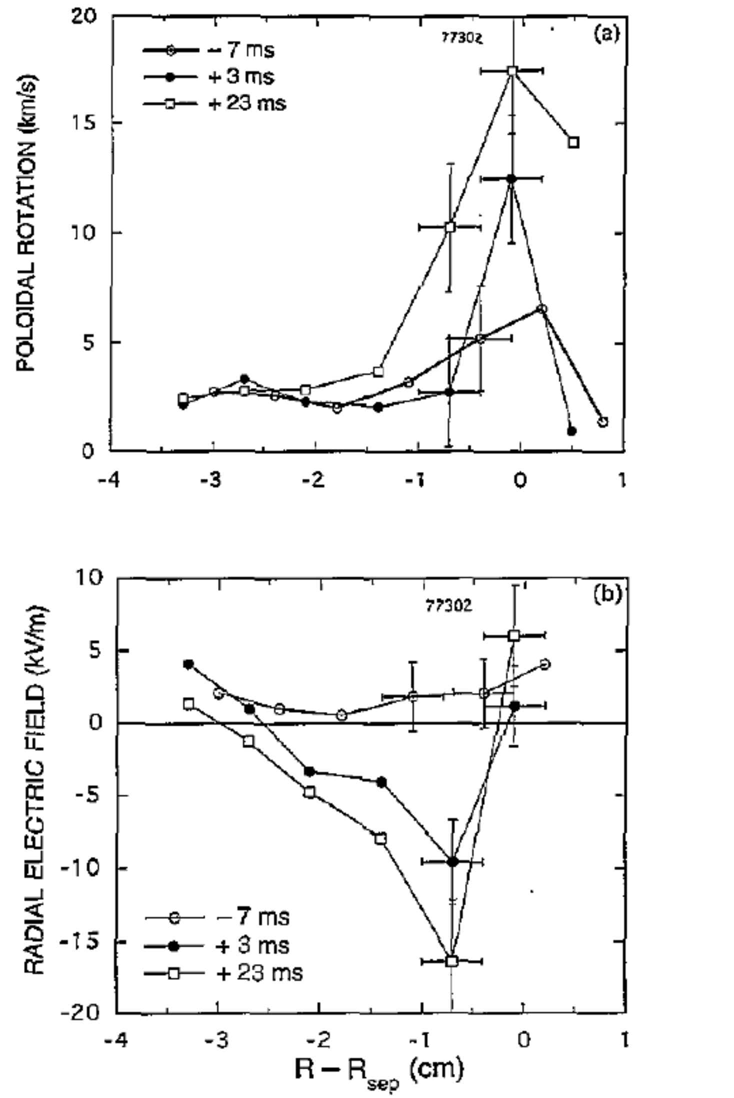
# THE PROFILE CHANGES ARE SIMILAR TO MAIN ION CER DATA

Profile changes during LCO cycle
**high** and **low** turbulence times

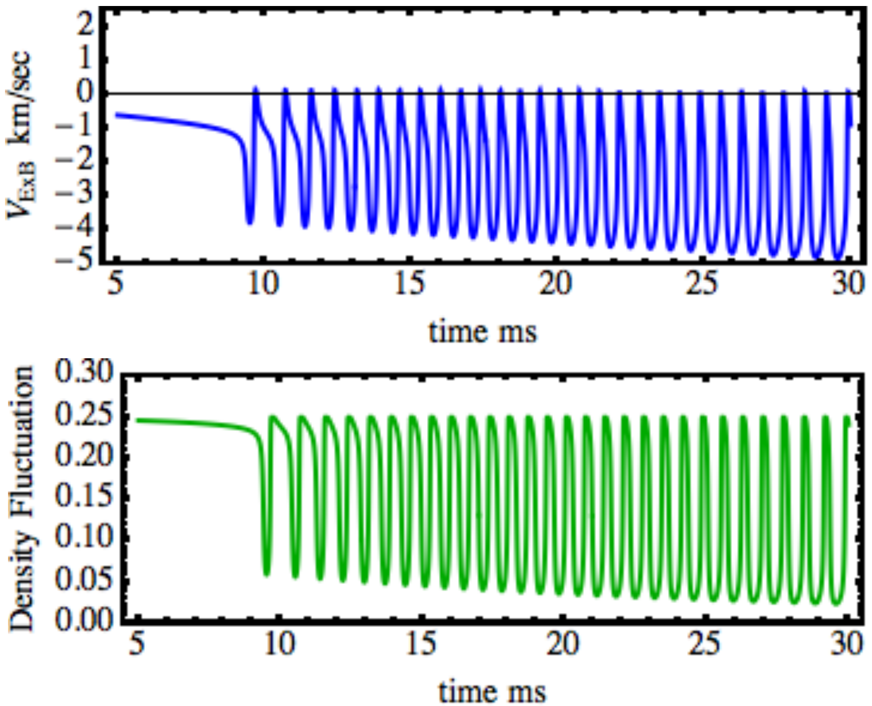



**1994 J. Kim et. al, PPCF 36, A183**

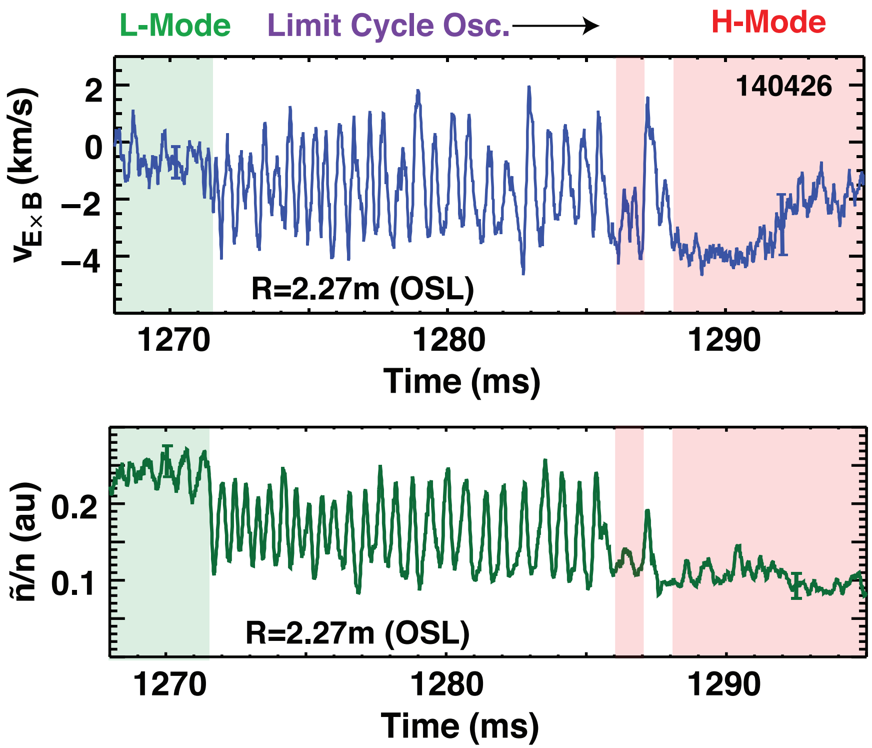


# LCO SOLUTIONS REASONABLY FIT THE DBS MEASUREMENTS

- **Using the DIII-D data for 140426 at 1265ms and R=2.265 determines:**
  - $a_{per}$= 0.215, $c_{neo}$= 0.2, $d_{par}$=$d_{tor}$= 10$m^2$/s, $d_{neo}$= 0.01$d_{par}$, $n_{pol}$= 632/s (NCLASS)
- **The LCO frequency of 1.6 KHz is fit for:** $b_{tor}$= 0.1, $b_{par}$= 0.17, $D_r$= 1.8cm
- **The ExB velocity is fit for:** $\gamma = \gamma_0 = 0.766 \times 10^5$ rad/s




2014 G.M. Staebler and R.J. Groebner, PPCF 57, 014025



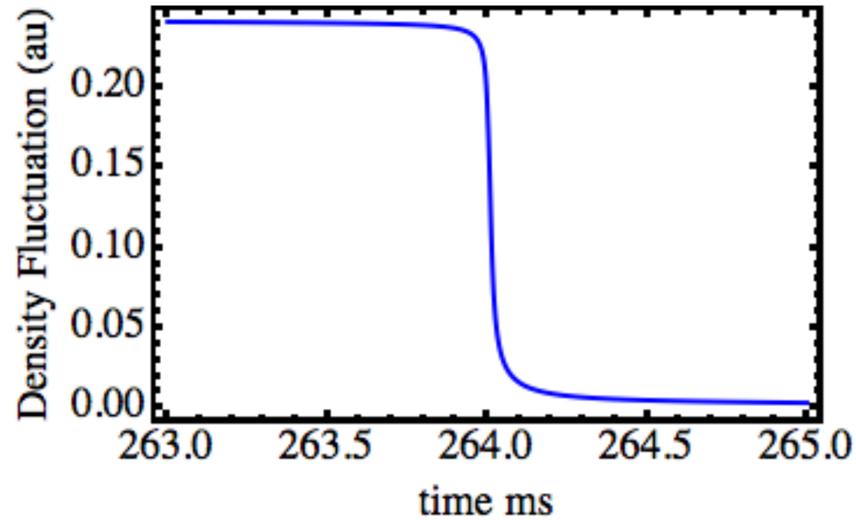

2012 L. Schmitz, PRL 108, 155002

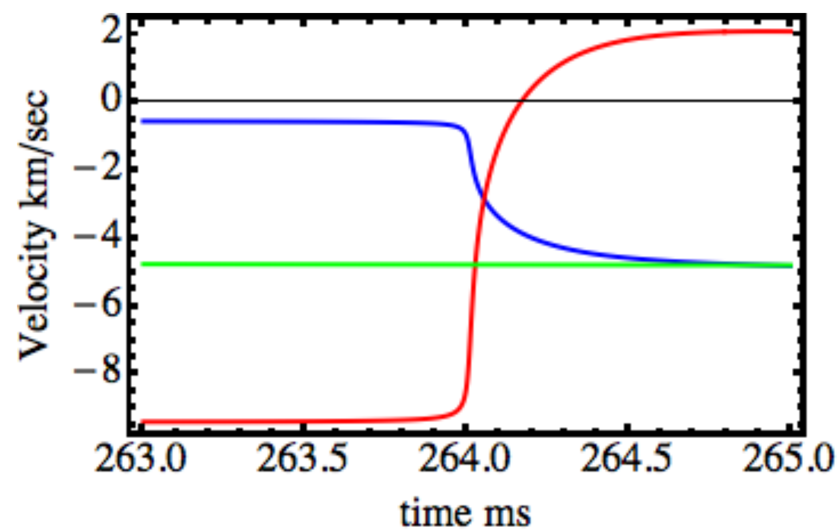
# ONE STEP L/H TRANSITIONS MATCH THE OBSERVED TRIGGER

- **Reducing the gradient scale length $\Delta_r$ from 1.8 cm to 1.5 cm gives one-step L/H transitions.**
- **The density fluctuation amplitude drops in ~0.1ms in agreement with measurements**

  1991 E.J. Doyle, Phys. Fluids B 3, 2300.
- **The ExB and vertical velocities rearrange in less than 0.5ms.**
  - **This timescale agrees with the observed poloidal velocity "trigger" for the L/H transition**

  1996 K.H. Burrell, PPCF 38, 1313
- **The diamagnetic velocity is externally controlled in this model but is observed to evolve more slowly than the poloidal velocity in experiment.**




2014 G.M. Staebler and R.J. Groebner, PPCF 57, 014025

# PROGRESS TOWARDS A PREDICTIVE THEORY OF THE LH-TRANSITION

- **After 27 years of active research by the fusion community a predictive model of the LH-transition is close at hand.**
  - ✓ Mean field ExB shear suppression of turbulence is well understood and modeled by the spectral-shift model
  - ✓ The importance of the velocities in the boundary layer at the separatrix is understood – modeling of ion orbit loss, charge exchange damping and 3D magnetic effects is needed.
  - ✓ 2D momentum transport reproduces both the trigger event and the dithering time scales which are much slower than decorrelation rate.
- **The last remaining major obstacle to computing the LH-threshold power is the shortfall in the gyrokinetic predicted L-mode energy transport.**
  - ✓ A solution to the L-mode shortfall problem has been demonstrated with multi-scale gyrokinetic simulations of C-MOD.
    - ✓ N. Howard and C. Holland, to be published in Nuclear Fusion.
  - ✓ A new model of zonal flow saturation of turbulence based on these multi-scale simulations has been developed.

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